

Power correlations in cosmology: limits on primordial non-Gaussian density fields.

A.J. Stirling¹ and J.A. Peacock²

¹*Institute for Astronomy, University of Edinburgh, Blackford Hill, Edinburgh EH9 3HJ*

²*Royal Observatory, Blackford Hill, Edinburgh EH9 3HJ*

ABSTRACT

We probe the statistical nature of the primordial density field by measuring correlations between the power in adjacent Fourier modes. For certain types of non-Gaussian field, these k -space correlations would be expected to be more extended than for a Gaussian field, providing a useful discriminatory test for Gaussian fields. We apply this test to the combined QDOT and 1.2-Jy IRAS galaxy survey and find the observed density field to be in good agreement with having Gaussian density fluctuations for modes with $k \lesssim 0.1 h \text{ Mpc}^{-1}$. From this result we are able to set quantitative limits on a class of possible non-Gaussian distributions – the product of a Gaussian field with an independent stochastic field. The maximum sensitivity is to modulations of a Gaussian field with coherence scales of $\sim 30 h^{-1} \text{ Mpc}$ and the rms modulation on this scale cannot greatly exceed unity. We discuss the improvements to this limit likely to be set by future surveys.

1 INTRODUCTION

A number of theories now set out to explain the initiation of density perturbations in the early universe. Since most of these ideas invoke new physics, with the prediction of additional fundamental fields, it is of interest to any cosmologist to narrow down the alternatives. One testable prediction that separates these theories is the statistics of the initial perturbations – some predict these to be non-Gaussian in form (most topological defect theories, and some versions of inflation), and others predict that the density field should be Gaussian (most types of inflation).

Previous work in this area has concentrated mainly on three statistical approaches: (1) comparing the moments of the density probability distribution function found from galaxy redshift surveys with those predicted from a Gaussian distribution, (e.g. Saunders et al. 1991; Gatzaga 1992; Nusser et al. 1995); (2) using topology to estimate the genus for the observed density-field contours (for instance Coles et al. 1993, Gott et al., Moore et al. 1992; Park et al. 1992; Vogeley et al. 1994); (3) using N-body simulations to compare the evolution and clustering properties of galaxies, starting from different non-Gaussian and Gaussian initial conditions (e.g. Moscardini et al. 1991, Weinberg & Cole 1992.)

Much of the work on higher order moments has concentrated on measuring statistics of the density field in the quasi-linear régime, and comparing the observed non-Gaussian characteristics with what would be expected for the gravitational evolution of initially Gaussian fluctuations. In the above analyses the density field has been found to be consistent with a distribution that was initially Gaussian. However, in all this work it is necessary to smooth the galaxy distribution with some filter, averaging the effect of

different regions of space. The central limit theorem then suggests that there is the danger that the appearance of Gaussian statistics will always be produced, whatever the underlying distribution.

An alternative approach adopted by Feldman, Kaiser, and Peacock (1994; FKP) is to look at the density field in Fourier space, where modes are separated out on the basis of scale, making it easy to probe the linear régime. FKP proposed two possible tests for a Gaussian density field. Both make use of the property that at high resolution in k space, independent Fourier modes have power values that fluctuate about the mean power for that particular scale. The first test was to look at the one-point distribution of these power fluctuations, which is predicted to be exponentially distributed in the Gaussian case. FKP found that for the QDOT 1-in-6 survey, the power distribution was in good agreement with the Gaussian prediction. However, Fan & Bardeen (1995) have subsequently argued that such one-point pdf tests have little discriminatory power: for a large enough sample, most types of non-Gaussian field are also expected to have an exponential distribution as a consequence of the central limit theorem. The second test suggested by FKP was to look at correlations between the power fluctuations which, for a Gaussian field, will be a function of the selection function only. This is less susceptible to the swamping effects of the central limit theorem, and it is the test that we investigate in this paper.

2 POWER CORRELATIONS

The general idea behind looking for correlations between power fluctuations is that for any infinite density field the

Fourier modes are independent i.e.:

$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2}^* \rangle = (2\pi)^3 P(k_1) \delta_D(\mathbf{k}_1 - \mathbf{k}_2), \quad (1)$$

where $\delta_D(\mathbf{k}_1 - \mathbf{k}_2)$ is the Dirac delta function and angle brackets here, and throughout the rest of this paper denote the ensemble average. For a finite sample of the density field, however, this mode independence is lost. This is because the finite region is effectively a product of the true infinite field, δ^{infinite} , and a mask, m . In Fourier space the observed density modes $\delta_{\mathbf{k}}^{\text{observed}}$ are then a convolution of the ‘true’ density modes, with the Fourier transform of the mask.

$$\delta_{\mathbf{k}}^{\text{observed}} = \delta_{\mathbf{k}}^{\text{infinite}} \star m_k \quad (2)$$

where $\delta_{\mathbf{k}}$ is given by:

$$\delta_{\mathbf{k}} = \int \delta(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d^3\mathbf{r}, \quad (3)$$

and $\delta(\mathbf{r}) \equiv [\rho(\mathbf{r}) - \bar{\rho}]/\bar{\rho}$ is the overdensity at position \mathbf{r} . This has the effect of mixing information between the modes over a scale comparable to one over the k -space width of the mask. Now, for a Gaussian field, the following relation applies (see FKP appendix B):

$$\begin{aligned} |\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2}^* \rangle|^2 &= \langle \hat{P}(\mathbf{k}_1) \hat{P}(\mathbf{k}_2) \rangle - P(\mathbf{k}_1) P(\mathbf{k}_2) \\ &= \langle \delta P(\mathbf{k}_1) \delta P(\mathbf{k}_2) \rangle \end{aligned} \quad (4)$$

where $P(k)$, the ensemble-average power, comes from equation (1); $\hat{P}(\mathbf{k})$ is the power in a single mode:

$$\hat{P}(\mathbf{k}) = |\delta_{\mathbf{k}}|^2; \quad (5)$$

and $\delta P(\mathbf{k}) = \hat{P}(\mathbf{k}) - P(k)$. This relation is dependent on two properties of Gaussian fields – a normal distribution for the one-point pdf, and the independence of the Fourier modes (i.e. the modes should have uncorrelated phases). For a finite sample of a Gaussian field, there will thus be correlations between the different power modes, with a two-point function for the power fluctuations which is the square of the two-point function for the Fourier amplitudes. For non-Gaussian fields in which intrinsic phase correlations exist, this proof does not apply, and we consider in Section 4 specific non-Gaussian models in which the power correlations are broader than the Gaussian prediction. This does not prove that all conceivable non-Gaussian fields could be detected in this way, but it does show that the correlations of the fluctuating power field give a necessary condition for the field to be considered Gaussian. Figure 1 shows how the measured field appears for the IRAS-galaxy data described in Section 3.

Quantitatively, we can define a power correlation function, ξ_P as follows:

$$\xi_P(\Delta\mathbf{k}) = \frac{\langle [\hat{P}(\mathbf{k}) - P(k)] [\hat{P}(\mathbf{k} - \Delta\mathbf{k}) - P(k - \Delta k)] \rangle}{\langle [\hat{P}(\mathbf{k}) - P(k)]^2 \rangle} \quad (6)$$

This is simply $|\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2}^* \rangle|^2$ as in equation (4), but normalised so that $\xi_P(0) = 1$. We now show how to evaluate this function for practical datasets.

For a galaxy survey FKP defined the ‘weighted galaxy fluctuation field’, $\delta(\mathbf{r})$ as

$$\delta(\mathbf{r}) \equiv \frac{w(\mathbf{r})[n_g(\mathbf{r}) - \alpha n_s(\mathbf{r})]}{[\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})]^{1/2}} \quad (7)$$

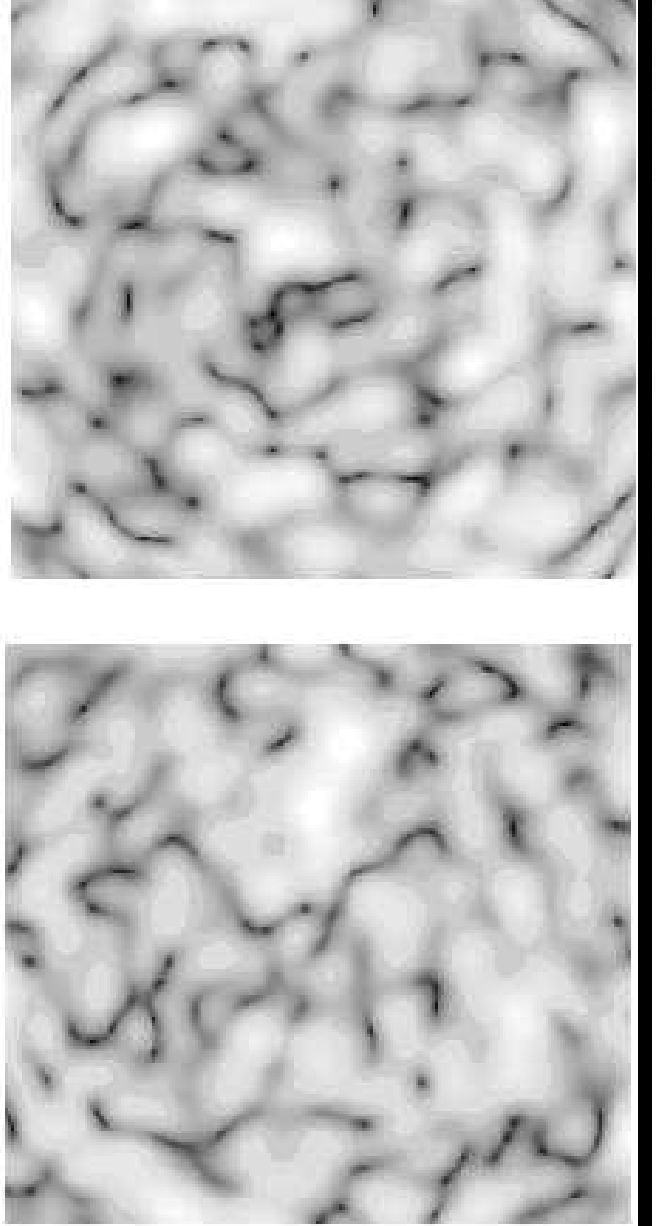


Figure 1. Illustrating the fluctuating power field from the IRAS data described in Section 3. Equal area projections of power from shells of constant $|\mathbf{k}|$ are shown (Figure 1a at $k = 0.1 h \text{ Mpc}^{-1}$, and figure 1b at $k = 0.15 h \text{ Mpc}^{-1}$). Black corresponds to low, and white to high power. The panels have width $\Delta k = 0.25 h \text{ Mpc}^{-1}$. While the power is not constant for a particular value of $|\mathbf{k}|$, there is a certain coherence scale over which there are significant correlations in power. As expected, this coherence scale does not appear to change with differing values of $|\mathbf{k}|$, and it forms the basis for a test for Gaussianity.

where $n_g(\mathbf{r}) = \sum_i \delta_D(\mathbf{r} - \mathbf{r}_i)$ with \mathbf{r}_i being the galaxy position vectors. The smooth background density is subtracted via the analogous number density n_s , which applies to a synthetic catalogue with number density $1/\alpha$ times that of the real catalogue. $\bar{n}(\mathbf{r})$ is the expected mean density of galaxies given the angular and luminosity selection criteria, and $w(\mathbf{r})$ is a weighting function designed to minimise the variance in power by favouring distant galaxies while the shot noise is

not a dominant contributor to the power. We have used the weighting function from FKP, which is:

$$w(\mathbf{r}) = [1 + \bar{n}(\mathbf{r})P(k)]^{-1}. \quad (8)$$

If this survey has an underlying Gaussian distribution, then FKP showed that the power correlation function would be of the form:

$$\xi_P(\Delta\mathbf{k}) = \frac{|P(k)Q(\Delta\mathbf{k}) + S(\Delta\mathbf{k})|^2}{[P(k) + S(0)]^2}, \quad (9)$$

provided the mean power, P can be taken as constant over the width of Q . This expression contains two functions of the mean density:

$$Q(\mathbf{k}) \equiv \frac{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})} \quad (10)$$

and

$$S(\mathbf{k}) \equiv \frac{(1 + \alpha) \int d^3r \bar{n}(\mathbf{r}) w^2(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}}{\int d^3r \bar{n}^2(\mathbf{r}) w^2(\mathbf{r})}. \quad (11)$$

The idea now is to estimate ξ_P via equation (6) and to compare this with the Gaussian prediction quoted in equation (9).

3 APPLICATION TO IRAS GALAXIES

3.1 Method

We have found the power correlation function, ξ_P for a combined dataset of ~ 4500 IRAS galaxies, consisting of the QDOT 1-in-6 0.6-Jy redshift survey (see Efsthathiou et al., 1990; Lawrence et al. 1996) and the Berkeley 1.2-Jy redshift survey (Fisher et al. 1993, Fisher et al. 1995). The power in each Fourier mode, ($= |\delta_{\mathbf{k}}|^2$) was found following the procedure described in FKP (see also Tadros & Efsthathiou 1995). We have used a direct Fourier transform of the survey, choosing ~ 6000 random orientations in the k -space shell over which to perform the integration. This was to sample power modes close enough together in k space to determine the shape of the correlation function accurately. For an estimator of $P(k)$, we have averaged $\hat{P}(\mathbf{k})$ over these different orientations in the k -space shell:

$$P(k) \simeq \frac{1}{M N} \sum_{i=1}^M \sum_{j=1}^N \hat{P}(k, \theta_i, \phi_j). \quad (12)$$

ξ_P was then evaluated using the assumption that the ensemble average can be replaced by an average over different orientations in k space. This assumption holds for a Gaussian field, which is the hypothesis we will be testing against. There is a small amount of anisotropy introduced by the angular part of the mask, so in comparing $\xi_P(\Delta\mathbf{k})$ with its theoretical prediction, we have also averaged Q , and S over the different orientations in the k -space shell.

The normalisation of ξ_P depends on the correct evaluation of the mean power, P , which can lead to problems if there are not enough independent patches in k space to get an accurate estimate. This can be overcome by evaluating ξ_P on a shell of fixed $|\mathbf{k}|$, for which the mean power is a constant. Then by requiring that $\xi_P \rightarrow 0$ for large $\Delta\mathbf{k}$, we

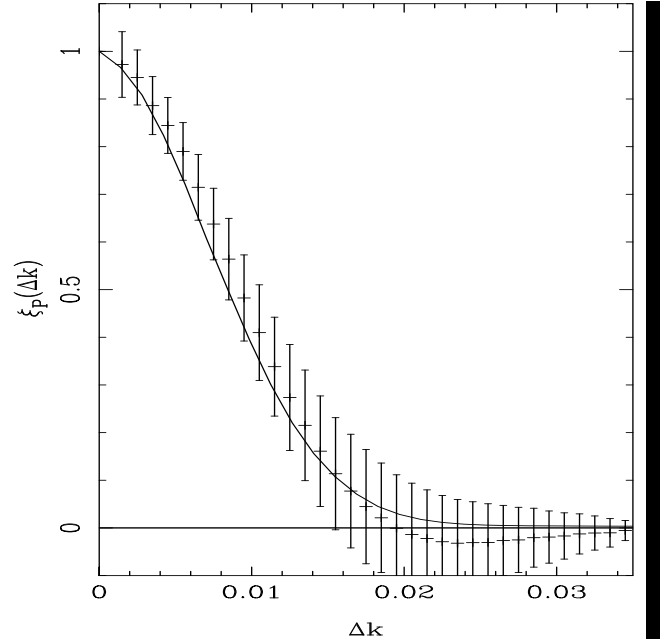


Figure 2. The power correlation function for the QDOT + 1.2-Jy survey. The solid line is the theoretical prediction for this survey if the underlying distribution is Gaussian. Error bars are based on Monte Carlo simulations of the deviations from the theoretical form for Gaussian surveys of the same size as the QDOT + 1.2-Jy.

can find the actual value for P , and renormalise accordingly.

The values of $|\mathbf{k}|$ used were chosen experimentally using the criteria that the scales probed should be in the linear régime, but not so large that shot noise dominates the signal. The k -space shell also needed to have large enough radius for there to be enough independent coherent patches over which to find an average power correlation function.

The galaxy coordinates that we use are in redshift space, and so peculiar velocities may affect the predicted shape of ξ_P . Empirically we find this to be a small effect, in the sense that ξ_P appears to be the same the radial and transverse directions, and we will examine it in detail elsewhere.

3.2 Results

Figure 2 is a plot of the power correlation function for the QDOT and 1.2-Jy survey with the prediction for an underlying Gaussian distribution. The error bars are based on expected deviations from the curve for a catalogue that has an underlying Gaussian distribution. These have been determined by making real space Monte Carlo realisations of the galaxy catalogue, and finding the spread in the shape of the correlation functions. The realisations are generated by Poisson sampling from a given underlying probability distribution, giving all catalogues the same selection function, angular mask, and power spectrum as the real catalogue, with a Gaussian distribution on large scales. The density fluctuations are generated as lognormal realisations (Coles & Jones 1991) to give non-linear features on smaller scales.

The figure shows that the QDOT+1.2-Jy survey appears to lie within the error bars predicted for an initial Gaussian distribution. Quantitatively, the goodness of fit can be characterised by performing a χ^2 -type test on the

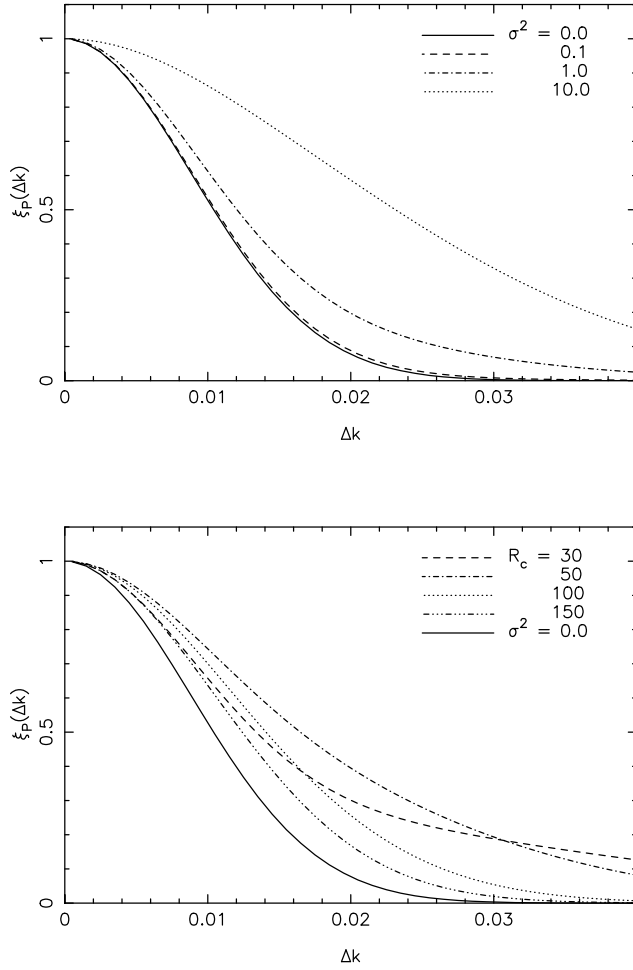


Figure 3. Predicted shapes of the correlation function for a product of Gaussian field: $\delta(\mathbf{r}) = \delta_G(\mathbf{r})[1 + \eta(\mathbf{r})]$ for varying forms of the power spectrum of η , $\langle |\eta_{\mathbf{k}}|^2 \rangle$. In the upper graph, the modulating scale, R_c is fixed at $50 h^{-1}$ Mpc, and the different lines represent different values of σ^2 : From the bottom up, the lines are for (1) zero amplitude of modulation (i.e. a Gaussian field); (2) $\sigma^2 = 0.1$; (3) $\sigma^2 = 1.0$; and (4) (the uppermost) $\sigma^2 = 10.0$. The lower graph has a fixed amplitude, σ^2 of 3.0, with varying modulating scales. From the bottom up, line (1) is for the Gaussian prediction as in the graph above; (2) $R_c = 150 h^{-1}$ Mpc; (3) the dashed line is for $R_c = 30 h^{-1}$ Mpc; (4) $R_c = 100 h^{-1}$ Mpc; and (5) (the uppermost line) for $R_c = 50 h^{-1}$ Mpc.

data:

$$\psi^2 = \sum_i \frac{(o_i - e_i)^2}{\sigma_i^2}, \quad (13)$$

where o_i is the set of observed data points, e_i is the equivalent set expected for a Gaussian distribution, and σ_i is the standard deviation for the observed data points, which we have taken to be the same as that found for the Gaussian case from the Monte Carlo simulations. ψ^2 does not follow the conventional χ^2 distribution, since the data points themselves are correlated, making the actual number of degrees of freedom fewer than the number of data points.

The probability distribution for ψ^2 has been determined from a frequency plot of ψ^2 values found from the 100 Monte Carlo simulated galaxy catalogues mentioned above. We find

a probability, $p(\psi^2 > \psi_{\text{measured}}^2)$ of 63%, (for the null hypothesis that the data set follows the Gaussian prediction). This is in good agreement with the Gaussian prediction, but how does it compare with what any non-Gaussian model might predict?

4 LIMITS ON NON-GAUSSIAN MODELS

Since the power correlation function for the Gaussian case is a function of the selection function only, one particular class of non-Gaussian models can be analysed with relative ease. This class is the product of a Gaussian field with another stochastic field, uncorrelated with the former:

$$\delta(\mathbf{r}) = \delta_G(\mathbf{r})[1 + \eta(\mathbf{r})] \quad (14)$$

This has been suggested as a possible alternative form for the density field, both on empirical grounds by Peebles (1983), and in some inflationary scenarios involving multiple scalar fields (e.g. Yi & Vishniac 1993). Locally, this field looks Gaussian, but on larger scales it is modulated so that the amplitude of fluctuations varies in different parts of space giving rise to quiet and noisy regions. This is equivalent to a field with zero skewness, but non-zero kurtosis. The one-point pdf is a simple test for this form of non-Gaussian behaviour; however a simple transformation of the field would restore a Gaussian looking pdf, but it would still be modulated in a way which our method could detect.

We can predict the form of ξ_P by treating the $(1 + \eta)$ part of the field as part of the selection function, so that

$$\bar{n}w \rightarrow \bar{n}w(1 + \eta) \quad (15)$$

in equations (10), and (11). Q then becomes:

$$Q' = Q + 2 Q \star \eta_{\mathbf{k}} + Q \star \eta_{\mathbf{k}} \star \eta_{\mathbf{k}}, \quad (16)$$

and S :

$$S' = S + S \star \eta_{\mathbf{k}} \quad (17)$$

We can now use equation (9) to obtain the modified correlation function:

$$\xi'_P(\Delta \mathbf{k}) = \left\langle \frac{|P(k)Q'(\Delta \mathbf{k}) + S'(\Delta \mathbf{k})|^2}{[P(k)Q'(0) + S'(0)]^2} \right\rangle \quad (18)$$

Substituting in the terms for Q' and S' from equations (16) and (17), and setting $PQ'(0) + S'(0) \equiv P'$ for ease of notation, this becomes:

$$\begin{aligned} P'^2 \xi'_P(\Delta \mathbf{k}) = & \langle |PQ + S|^2 \rangle \\ & + \langle |(2PQ + S) \star \eta_{\mathbf{k}} + PQ \star \eta_{\Delta \mathbf{k}} \star \eta_{\Delta \mathbf{k}}|^2 \rangle \\ & + \langle (PQ + S) [(2PQ + S) \star \eta_{\Delta \mathbf{k}} + PQ \star \eta_{\Delta \mathbf{k}} \star \eta_{\Delta \mathbf{k}}]^* \\ & + \text{conj.} \rangle \end{aligned} \quad (19)$$

Now, by noting that odd powers in $\eta_{\mathbf{k}}$ average to zero, and that $\langle |f \star \eta_{\mathbf{k}}|^2 \rangle = \langle |f|^2 \rangle \star \langle |\eta_{\mathbf{k}}|^2 \rangle$, where f is a function uncorrelated with η , we can rewrite equation (19) as:

$$\begin{aligned} P'^2 \xi'_P(\Delta \mathbf{k}) = & \xi_P^G + \langle |2P(k)Q + S|^2 \rangle \star \langle |\eta_{\Delta \mathbf{k}}|^2 \rangle \\ & + \langle |P(k)Q \star \eta_{\Delta \mathbf{k}} \star \eta_{\Delta \mathbf{k}}|^2 \rangle \\ & + \langle (P(k)Q + S)[P(k)Q \star \eta_{\Delta \mathbf{k}} \star \eta_{\Delta \mathbf{k}}]^* + \text{conj.} \rangle \end{aligned} \quad (20)$$

and this can be written as:

$$\begin{aligned}
P'^2 \xi'_P(\Delta \mathbf{k}) &= \xi_P^G(\Delta \mathbf{k}) \\
&+ \langle |2P(k)Q(\Delta \mathbf{k}) + S(\Delta \mathbf{k})|^2 \rangle \star \langle |\eta_{\Delta \mathbf{k}}|^2 \rangle \\
&+ P(k)^2 \langle |Q(\Delta \mathbf{k})|^2 \rangle \star \langle |\eta_{\Delta \mathbf{k}}|^2 \rangle \star \langle |\eta_{\Delta \mathbf{k}}|^2 \rangle \\
&+ 2 \langle \text{Re}\{P(k)Q(\Delta \mathbf{k}) [P(k)Q(\Delta \mathbf{k}) + S(\Delta \mathbf{k})]^*\} \rangle \times \\
&\int \langle |\eta_{\mathbf{k}}|^2 \rangle d\mathbf{k},
\end{aligned} \tag{21}$$

where ξ_P^G is the Gaussian contribution to the power correlation function. The effect of η is to broaden the correlation function by an amount dependent on the power spectrum of the modulating field, thus acting like an unaccounted for part of the selection function. Since this non-Gaussian field is a product of a Gaussian with a modulating field, and the modulating part can be treated as part of the selection function, we are still justified in our use of the approximation in equation (12).

Figure 3 shows different shapes of the power correlation function, for a range of power spectra for η . We have parameterised this power spectrum by supposing that the stochastic process, η , is white noise with an amplitude σ^2 , and a Gaussian cut-off on scales R_c , so that

$$P_\eta = \langle |\eta_{\mathbf{k}}|^2 \rangle = (2\sqrt{\pi} R_c)^3 \sigma^2 \exp[-k^2 R_c^2] \tag{22}$$

and

$$\frac{1}{(2\pi)^3} \int P_\eta d^3 \mathbf{k} = \sigma^2 \tag{23}$$

For the simple case of a small amplitude modulation in the absence of shot noise, equation (21) reduces to:

$$\begin{aligned}
\langle |Q'(\Delta k)|^2 \rangle &= \langle |Q + 2Q \star \eta_{\mathbf{k}}|^2 \rangle \\
&= \exp[-\Delta k^2 / 4\alpha^2] \\
&+ \frac{32\alpha^3 R_c^3 \sigma^2}{(1 + 4\alpha^2 R_c^2)^{3/2}} \exp[-\Delta k^2 R_c^2 / (4\alpha^2 R_c^2 + 1)]
\end{aligned} \tag{24}$$

where α is the effective k -space Gaussian size of the survey. This expression is illustrated in figure 3.

We are now in a position to constrain the amplitude and scale of modulation that could possibly be present in the observed density field. We have the measured value of ψ^2 for the QDOT+1.2Jy survey from the Gaussian prediction, and can specify the largest allowed value of ψ^2 at a given level of confidence. For a given deviation, we can find a set of values of σ and R_c for the power spectrum of η which would give this level of departure from the Gaussian prediction. These contours are shown in figure 4. Present data allow $\sigma \sim 3$ modulation on $\sim 30 h^{-1}$ Mpc scales at the 95% level.

It is clear from figure 4 that the sensitivity to modulations is peaked around scales of $\sim 30 h^{-1}$ Mpc. From figure 3, we see that this scale is set by the width of ξ_P for the unmodulated field. Both larger and smaller scales give a smaller perturbation to ξ_P ; in the former case because there are too few independent samples within the sample volume. The latter case is more subtle: although modulations characterised by a small length scale give rise to broad power correlations, the amplitude of the modulation is small compared with the amplitude of the Gaussian field. So fluctuations in the power modes persist for small Δk – as with a Gaussian field, and the effect of the modulation is only seen at large Δk . We are

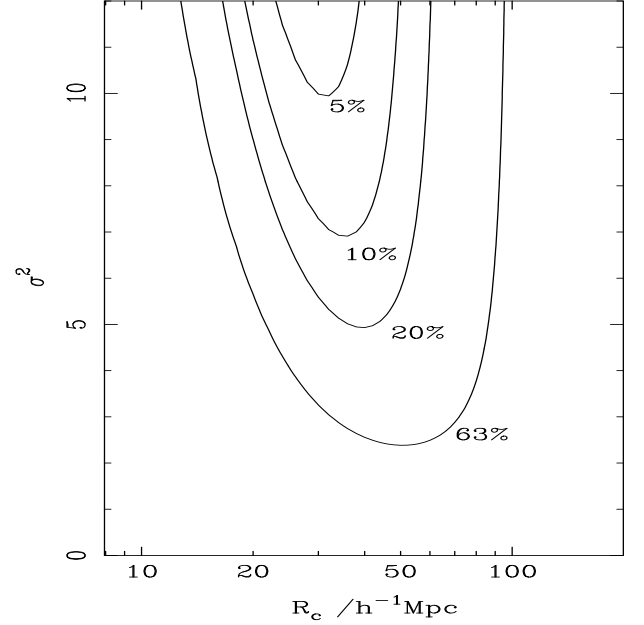


Figure 4. Contour plot of ψ^2 deviations in $R_c - \sigma^2$ parameter space. The contours represent different ψ^2 values for the deviation of ξ_P from the Gaussian theoretical prediction. The outermost contour is the measured deviation, ψ^2 of ξ_P for the QDOT+1.2-Jy survey from the Gaussian form. This corresponds to a probability that $\psi^2 > \psi^2_{\text{measured}}$ of 63% if the survey is from a Gaussian distribution. The innermost contour indicates the region of parameter space for $\langle |\eta_{\mathbf{k}}|^2 \rangle$ that can be ruled out with 95% confidence as deduced from the form of ξ_P for the QDOT+1.2-Jy, and the corresponding error bars.

least sensitive to large Δk correlations because of the need to renormalise the correlation function due to the uncertainty in the mean power, P (see Section 3.1).

5 SUMMARY AND OUTLOOK

This preliminary exploration suggests that the use of power correlations can be a useful tool in allowing quantitative limits to be placed on certain types of non-Gaussian fields. Future work will include examining in more detail the effects of redshift distortions, and extrapolating the technique for use in larger redshift surveys. Since the error bounds scale roughly as volume $^{-1/2}$, we can expect to tighten the limits on σ by about one order of magnitude with the next generation of redshift surveys. We should also be able to increase the sensitivity of this test to modulations on scales approaching $600 h^{-1}$ Mpc. It will be interesting to compare the power of this test with alternative methods, such as the use of topology to detect non-Gaussian signatures.

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